OCR
RECOGNISING ACHIEVEMENT

Monday 28 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The differential equation

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = \sin x$$

is to be solved.

(i) Show that 2 is a root of the auxiliary equation. Find the other two roots and hence find the general solution of the differential equation. **[10]**

When $x = 0$, $y = 1$ and $\frac{dy}{dx} = 0$. Also, y is bounded as $x \to \infty$.

(ii) Find the particular solution. **[6]**

(iii) Write down an approximate solution for large positive values of x. Calculate the amplitude of this approximate solution and sketch the solution curve for large positive x. **[4]**

Suppose instead that a solution is required that is bounded as $x \to -\infty$.

(iv) Determine whether there is a solution for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$. **[4]**

2 A ball of mass m kg falls vertically from rest through a liquid. At time t s, the velocity of the ball is $v\,\mathrm{m\,s^{-1}}$ and the ball has fallen a distance x m. The forces on the ball are its weight and a total upwards force of R N. A student investigates three models for R.

In the first model $R = mkv$, where k is a positive constant.

(i) Show that $\frac{dv}{dt} = 9.8 - kv$ and hence find v in terms of t and k. **[7]**

The terminal velocity of the ball is observed to be $7\,\mathrm{m\,s^{-1}}$.

(ii) Find k. **[1]**

In the second model, $R = 0.2mv^2$.

(iii) Find v in terms of t. Show that your solution is consistent with a terminal velocity of $7\,\mathrm{m\,s^{-1}}$. **[10]**

In the third model, $R = 0.529mv^{\frac{3}{2}}$. Euler's method is to be used to solve for v numerically.

The algorithm is given by $t_{r+1} = t_r + h$, $v_{r+1} = v_r + h\dot{v}_r$ with $(t_0, v_0) = (0, 0)$.

(iv) Show that $\frac{dv}{dt} = 9.8 - 0.529v^{\frac{3}{2}}$ and find v when $t = 0.2$ using Euler's method with a step length of 0.1. **[5]**

(v) Show that this model is consistent with a terminal velocity of approximately $7\,\mathrm{m\,s^{-1}}$. **[1]**

3 **(a)** Solve the differential equation

$$\frac{dy}{dx} - y\tan x = \sin x$$

to find y in terms of x subject to the condition $y = 1$ when $x = 0$. **[9]**

(b) Consider the differential equations

$$\frac{dy}{dx} + f(x)y = g(x), \qquad (1)$$

$$\frac{dy}{dx} + f(x)y = 0. \qquad (2)$$

Show that if $y = p(x)$ satisfies (1) and $y = c(x)$ satisfies (2), then $y = p(x) + Ac(x)$ satisfies (1), where A is an arbitrary constant. **[5]**

(c) The differential equation

$$\frac{dy}{dx} + \frac{2y}{x} = 2e^{x^2}\left(\frac{x^2+1}{x}\right) \qquad (3)$$

is to be solved.

(i) Verify that $y = e^{x^2}$ satisfies (3). **[3]**

(ii) Find the general solution of $\frac{dy}{dx} + \frac{2y}{x} = 0$, giving y in terms of x. **[4]**

(iii) Use the result of part **(b)** to find a solution of (3) for which $y = 1$ when $x = 1$. **[3]**

4 The simultaneous differential equations

$$\frac{dx}{dt} = -\frac{1}{2}x - \frac{3}{2}y + t$$

$$\frac{dy}{dt} = \frac{3}{2}x - \frac{1}{2}y + 2t$$

are to be solved.

(i) Eliminate y to obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[13]**

(ii) Find the corresponding general solution for y. **[4]**

When $t = 0$, $x = 1$ and $y = 0$.

(iii) Find the particular solutions. **[3]**

(iv) Show that in this case $x + y$ tends to a finite limit as $t \to \infty$ and state its value. Determine whether $x + y$ is equal to this limit for any values of t. **[4]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	
	(answer space continued on next page)

1 (i)	(continued)
1 (ii)	

1 (iii)	
1 (iv)	

2 (i)	
2 (ii)	

2 (iii)	

2 (iv)	
2 (v)	

3 (a)	

3 (b)	
3 (c) (i)	

3 (c) (ii)	
3 (c) (iii)	

4 (i)	
	(answer space continued on next page)

4 (i)	(continued)

4 (ii)	

4 (iii)	

4 (iv)	
	(answer space continued on next page)

4 (iv)	(continued)

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4758:** Differential Equations

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

Question			Answer	Marks	Guidance	
1	(i)		$\lambda^3+2\lambda^2-5\lambda-6=0$	M1		
			$2^3+2\times2^2-5\times2-6=0$	E1	Allow if implicit in factorisation of cubic	
			$(\lambda-2)(\lambda+1)(\lambda+3)=0$	M1	Attempt roots (any method)	
			$\lambda=(2),-1,-3$	A1		
			CF $A\mathrm{e}^{2x}+B\mathrm{e}^{-x}+C\mathrm{e}^{-3x}$	F1		
			PI $y=a\sin x+b\cos x$	B1		
			$y'=a\cos x-b\sin x \qquad y''=-a\sin x-b\cos x$			
			$y'''=-a\cos x+b\sin x$			
			$(-a\cos x+b\sin x)+2(-a\sin x-b\cos x)$ $-5(a\cos x-b\sin x)-6(a\sin x+b\cos x)=\sin x$	M1	Differentiate and substitute	
			$\left.\begin{array}{l}-6a-8b=0\\-8a+6b=1\end{array}\right\}\Rightarrow a=-\frac{2}{25},b=\frac{3}{50}$	M1 A1	Compare coefficients and solve	
			GS $y=\frac{3}{50}\cos x-\frac{2}{25}\sin x+A\mathrm{e}^{2x}+B\mathrm{e}^{-x}+C\mathrm{e}^{-3x}$	F1		
				[10]		
1	(ii)		bounded so $A=0$	F1		
			$x=0,y=1\Rightarrow 1=\frac{3}{50}+B+C$	F1	Use condition	
			$y'=-\frac{3}{50}\sin x-\frac{2}{25}\cos x-B\mathrm{e}^{-x}-3C\mathrm{e}^{-3x}$	M1	Differentiate	
			$x=0,y'=0\Rightarrow 0=-\frac{2}{25}-B-3C$	M1	Use condition	
			$B=\frac{29}{20},C=-\frac{51}{100}$	A1		
			$y=\frac{3}{50}\cos x-\frac{2}{25}\sin x+\frac{29}{20}\mathrm{e}^{-x}-\frac{51}{100}\mathrm{e}^{-3x}$	F1		
				[6]		
1	(iii)		$y\approx\frac{3}{50}\cos x-\frac{2}{25}\sin x$	F1		
			amplitude $=\frac{1}{50}\sqrt{3^2+4^2}=\frac{1}{10}$	M1 A1		
				B1	Sketch showing oscillations with their amplitude. More than one oscillation, ignore origin	
				[4]		

Question			Answer	Marks	Guidance	
1	(iv)		bounded so $B = C = 0$	B1		
			$x = 0, y = 1 \Rightarrow 1 = \frac{3}{50} + A \Rightarrow A = \frac{47}{50}$	M1		
			$x = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{25} + 2\left(\frac{47}{50}\right) \neq 0$	M1	Or $A = \frac{1}{25}$	
			So no such solution	A1 **[4]**	www	
2	(i)		N2L: $m\frac{\mathrm{d}v}{\mathrm{d}t} = 9.8m - mkv \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}t} = 9.8 - kv$	E1		
			EITHER $\int \frac{1}{9.8 - kv}\,\mathrm{d}v = \int \mathrm{d}t$	M1	Separate and integrate	
			$-\frac{1}{k}\ln\lvert 9.8 - kv\rvert = t + c$	A1 A1	LHS RHS (including constant on one side)	
			$9.8 - kv = A\mathrm{e}^{-kt}$	M1	Rearrange, dealing properly with constant	
			$t = 0, v = 0 \Rightarrow 9.8 = A$	M1	Use condition	
			$v = \frac{9.8}{k}\left(1 - \mathrm{e}^{-kt}\right)$	A1	cao	
			OR Integrating factor e^{kt}	M1		
			$v\mathrm{e}^{kt} = \int 9.8\mathrm{e}^{kt}\,\mathrm{d}t$	A1	Multiply both sides by IF and recognise derivative on LHS	
			$v\mathrm{e}^{kt} = \frac{9.8}{k}\mathrm{e}^{kt} + A$	M1	Integrate both sides	
				A1	Must include constant	
			$t = 0, v = 0 \Rightarrow A = -\frac{9.8}{k}$	M1	Use condition	
			$v = \frac{9.8}{k}\left(1 - \mathrm{e}^{-kt}\right)$	A1	cao	
			OR Auxiliary equation $\lambda + k = 0$	M1		
			CF $v = A\mathrm{e}^{-kt}$	A1		
			PI $v = b, v' = 0 \therefore b = \frac{9.8}{k}$	M1		

Question			Answer	Marks	Guidance	
			GS $v = Ae^{-kt} + \frac{9.8}{k}$	A1		
			$t = 0, v = 0 \Rightarrow A = -\frac{9.8}{k}$	M1	Use condition	
			$v = \frac{9.8}{k}\left(1 - e^{-kt}\right)$	A1	cao	
				[7]		
2	**(ii)**		$k = \frac{9.8}{7} = 1.4$	B1		
				[1]		
2	**(iii)**		$m\frac{dv}{dt} = 9.8m - 0.2mv^2$	M1		
			$\int \frac{1}{9.8 - 0.2v^2}\, dv = \int dt$	M1		
			$\int \frac{5}{49 - v^2}\, dv = t + c_2$	A1	RHS (including constant on one side)	
			$\frac{5}{14}\int\left(\frac{1}{7+v} + \frac{1}{7-v}\right)dv = t + c_2$	M1	Integrate	
			$\frac{5}{14}\left(\ln\lvert 7+v\rvert - \ln\lvert 7-v\rvert\right) = t + c_2$	A1	LHS	
			$\ln\left\lvert\frac{7+v}{7-v}\right\rvert = \frac{14}{5}\left(t + c_2\right)$			
			$\frac{7+v}{7-v} = Be^{14t/5}$	M1	Rearrange into a form without ln, dealing properly with constant	
			$t = 0, v = 0 \Rightarrow B = 1$	M1	Use condition	
			$7 + v = e^{14t/5}\left(7 - v\right)$	M1	Rearrange to get v in terms of t	
			$v = 7\left(\frac{e^{14t/5} - 1}{e^{14t/5} + 1}\right) = 7\left(\frac{1 - e^{-14t/5}}{1 + e^{-14t/5}}\right)$	A1	oe	
			as $t \to \infty, v \to 7\left(\frac{1-0}{1+0}\right) = 7$	E1		
				[10]		

Question			Answer	Marks	Guidance	
2	(iv)		$\dot{v} = g - 0.529v^{3/2}$	E1		
			t | v | $\dot{v}$ | $h\dot{v}$	M1	Use algorithm	
			0 | 0 | 9.8 | 0.98	A1	$v(0.1)$	
			0.1 | 0.98 | 9.2868 | 0.92868	A1	$\dot{v}(0.1)$ at least 3d.p.	
			0.2 | 1.9087	A1	$v(0.2) = 1.91$ to 3s.f.	
				[5]		
2	(v)		$\dot{v} = 0 \Rightarrow g = 0.529v^{3/2} \Rightarrow v = 7.00$ (3 sf)	E1	Or $9.8 - 0.529 \times 7^{3/2} \approx 0$	
				[1]		
3	(a)		$I = \exp\left(\int -\tan x \, dx\right)$	M1		
			$= \exp(-\ln\|\sec x\|)$ or $\exp(\ln\|\cos x\|)$	A1		
			$= \cos x$	A1		
			$\cos x \frac{dy}{dx} - y \sin x = \sin x \cos x$			
			$\frac{d}{dx}(y \cos x) = \sin x \cos x$	M1	Multiply and recognise derivative	
			$y \cos x = \int \sin x \cos x \, dx$	M1	Attempt integral	
			$= \int \frac{1}{2} \sin 2x \, dx$	M1	Use identity, substitution or inspection on RHS	
			$= -\frac{1}{4} \cos 2x + c_1$ (or $\frac{1}{2} \sin^2 x + k$)	A1	oe (but must include constant)	
			$x = 0, y = 1 \Rightarrow 1 = -\frac{1}{4} + c_1$	M1	Use condition	
			$y = \frac{5 - \cos 2x}{4 \cos x}$ or $y = \frac{\sin^2 x + 2}{2 \cos x}$ or $y = \frac{3 - \cos^2 x}{2 \cos x}$	A1	oe	
				[9]		

Question			Answer	Marks	Guidance	
3	(b)		$p'(x)+f(x)p(x)=g(x)$	M1	Must be $p'(x)$ oe	
			$c'(x)+f(x)c(x)=0$	M1	Must be $c'(x)$ oe	
			$\frac{dy}{dx}+f(x)y=p'(x)+Ac'(x)+f(x)\left(p(x)+Ac(x)\right)$	M1	Substitute in DE	
			$=p'(x)+f(x)p(x)+A\left(c'(x)+f(x)c(x)\right)$	M1	Separate p and c terms	
			$=g(x)+A\times 0=g(x)$	E1	Complete argument	
				[5]		
3	(c)	(i)	$y=e^{x^2}\Rightarrow\frac{dy}{dx}=2xe^{x^2}$	B1		
			so LHS of DE $=2xe^{x^2}+\frac{2}{x}e^{x^2}$	M1		
			$=2e^{x^2}\left(x+\frac{1}{x}\right)=2e^{x^2}\left(\frac{x^2+1}{x}\right)$	E1		
				[3]		
3	(c)	(ii)	$\frac{dy}{dx}=-\frac{2y}{x}\Rightarrow\int\frac{1}{y}dy=\int-\frac{2}{x}dx$	M1		
			$\ln\lvert y\rvert=-2\ln\lvert x\rvert+c_2$	A1	LHS	
				A1	RHS including constant	
			$y=Ax^{-2}$	A1	cao	
			OR Integrating factor = x^2	B1		
			$\frac{d}{dx}\left(yx^2\right)=0$	M1		
			$yx^2=A$	A1		
			$y=Ax^{-2}$	A1	cao	
				[4]		

Question			Answer	Marks	Guidance	
3	(c)	(iii)	$y = e^{x^2} + Ax^{-2}$	B1	Here A combines the arbitrary constants of **(b)** and **(c) (ii)** into a single arbitrary constant.	
			$1 = e^1 + A$	M1	Use condition	
			$y = e^{x^2} + (1-e)x^{-2}$	A1		
				[3]		
4	(i)		$y = \frac{2}{3}\left(-\dot{x} - \frac{1}{2}x + t\right)$	M1		
			$\dot{y} = \frac{2}{3}\left(-\ddot{x} - \frac{1}{2}\dot{x} + 1\right)$	M1	Differentiate	
			$\frac{2}{3}\left(-\ddot{x} - \frac{1}{2}\dot{x} + 1\right) = \frac{3}{2}x - \frac{1}{2}\cdot\frac{2}{3}\left(-\dot{x} - \frac{1}{2}x + t\right) + 2t$	M1	Substitute	
			$2\ddot{x} + 2\dot{x} + 5x = 2 - 5t$	A1	oe	
			AE $2\lambda^2 + 2\lambda + 5 = 0$	M1		
			$\lambda = -\frac{1}{2} \pm \frac{3}{2}\mathrm{j}$	A1		
			CF $e^{-\frac{1}{2}t}\left(A\cos\frac{3}{2}t + B\sin\frac{3}{2}t\right)$	M1	Correct form	
				F1	FT wrong roots	
			PI $x = a + bt$	B1		
			$\dot{x} = b,\ \ddot{x} = 0 \Rightarrow 2b + 5(a + bt) = 2 - 5t$	M1	Differentiate and substitute	
			$\left.\begin{array}{r} 2b + 5a = 2 \\ 5b = -5 \end{array}\right\} \Rightarrow a = \frac{4}{5}, b = -1$	M1 A1	Equate coefficients and solve	
			GS $x = \frac{4}{5} - t + e^{-\frac{1}{2}t}\left(A\cos\frac{3}{2}t + B\sin\frac{3}{2}t\right)$	F1		
				[13]		

Question			Answer	Marks	Guidance	
4	(ii)		$y=\frac{2}{3}\left(-\dot{x}-\frac{1}{2}x+t\right)$	M1		
			$\dot{x}=-1-\frac{1}{2}e^{-\frac{1}{2}t}\left(A\cos\frac{3}{2}t+B\sin\frac{3}{2}t\right)$ $+e^{-\frac{1}{2}t}\left(-\frac{3}{2}A\sin\frac{3}{2}t+\frac{3}{2}B\cos\frac{3}{2}t\right)$	M1 F1	Must be using product rule Must be GS from **(i)**	
			$y=\frac{2}{5}+t+e^{-\frac{1}{2}t}\left(A\sin\frac{3}{2}t-B\cos\frac{3}{2}t\right)$	A1		
				[4]		
4	(iii)		$x=1,t=0\Rightarrow 1=\frac{4}{5}+A\Rightarrow A=\frac{1}{5}$	M1		
			$y=0,t=0\Rightarrow 0=\frac{2}{5}-B\Rightarrow B=\frac{2}{5}$	M1		
			$x=\frac{4}{5}-t+e^{-\frac{1}{2}t}\left(\frac{1}{5}\cos\frac{3}{2}t+\frac{2}{5}\sin\frac{3}{2}t\right)$			
			$y=\frac{2}{5}+t+e^{-\frac{1}{2}t}\left(\frac{1}{5}\sin\frac{3}{2}t-\frac{2}{5}\cos\frac{3}{2}t\right)$	A1	Both	
				[3]		
4	(iv)		$x+y=\frac{6}{5}+e^{-\frac{1}{2}t}\left(\frac{3}{5}\sin\frac{3}{2}t-\frac{1}{5}\cos\frac{3}{2}t\right)$	M1	Adding and attempting the limit	
			$t\to\infty\Rightarrow e^{-\frac{1}{2}t}\to 0\Rightarrow x+y\to\frac{6}{5}$	E1	FT for finite limit	
			$x+y=\frac{6}{5}\Leftrightarrow\frac{3}{5}\sin\frac{3}{2}t-\frac{1}{5}\cos\frac{3}{2}t=0\Leftrightarrow\tan\frac{3}{2}t=\frac{1}{3}$	M1	Establish equation and indicate method	
			which occurs (infinitely often)	E1	Correctly investigate the existence of a solution, but explicit solution for t not required.	
				[4]		

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RECOGNISING ACHIEVEMENT

GCE

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Advanced GCE **A2 7895-8**

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OCR Report to Centres

January 2013

Oxford Cambridge and RSA Examinations

4758 Differential Equations (Written Examination)

General Comments

Candidates showed a sound understanding of the basic methods of solution of the types of differential equations covered by this specification. The presentation of solutions was usually of a good quality with clear explanations. As in previous series, Questions 1 and 4, on second and higher order differential equations, were chosen by almost all candidates. There was evidence that Question 3 was the next question of preference, but many candidates struggled with the first two parts and then opted for Question 2 instead. Others appeared to answer any parts of Questions 2 and 3 that they could, leaving the examiner to determine which of the two questions counted towards their total.

There were fewer fully correct solutions to questions than in previous series. There were more arithmetical errors than usual, particularly in the first parts of Questions 1 and 4. Candidates should be encouraged to work accurately even with work that is familiar. Although incorrect work is followed through and given credit, it is inevitable that errors early in a solution will lead to some loss of marks. In the case of second order linear differential equations an incorrect auxiliary equation, or the incorrect solution of an auxiliary equation, can lead to a different form of general solution and this may impact on the ease or feasibility of the requests in the later parts of a question.

Comments on Individual Questions

1 Third order linear differential equation

(i) Almost all candidates were able to use the information in the question to make a good attempt at solving the given third order differential equation. The complementary function was usually found accurately, but there were a surprising number of numerical errors when finding the particular integral.

(ii) The majority of candidates were successful in using the initial conditions to find a particular solution; it was pleasing to see that the less familiar form of condition, that y was bounded for very large values of x, was interpreted correctly.

(iii) Candidates seemed less confident in answering this part. A common error was to assume that the coefficient of either the sine or the cosine term was equal to the amplitude of the motion.

(iv) The majority of candidates were able to use the condition that y was bounded for very large negative values of x to deduce that two of the unknown coefficients in their particular solution from part (i) had to be zero. Many candidates then went on to apply the other two initial conditions to obtain two different values for the remaining coefficient, indicating that there was no consistent solution. A common error was to apply just one of the initial conditions to find the unknown coefficient and then state that this was a solution.

2 First order differential equations

(i) Almost all candidates applied Newton's second law to establish the given differential equation. They then proceeded to find the general solution either by the integrating factor method or by separating the variables or by finding the complementary function and particular integral. The three methods were equally popular and successful. A significant minority of candidates did not seem to realise that they needed to apply the initial condition, that the ball fell from rest, to find the value of their constant of integration.

(ii) This was almost always correct.

(iii) There were some excellent solutions to this part of the question with very clear accurate work and gaining full marks. However, the majority of candidates struggled to solve the differential equation. Separating the variables leads to an integral of the form $\int \frac{1}{a^2 - x^2} \mathrm{d}x$ and evaluating this proved to be a stumbling block for the majority. The most common attempts resulted in inverse tangents or in incorrect logarithmic terms of the form $\ln(a^2 - x^2)$. A denominator that is the difference of two squares is a common occurrence in problems of this type and candidates should be encouraged to be vigilant.

(iv) Euler's method is familiar territory for candidates and the majority of candidates were successful, presenting their working clearly.

(v) This was almost always correct.

3 First order differential equation

A significant number of candidates were not able to complete their attempts at the first two parts of this question. They chose to omit the remainder of the question and attempt another question instead.

(a) Almost all candidates recognised the method of solution as the integrating factor method and made a good start. Most candidates evaluated the integrating factor correctly as cos x, but they did not recognise a method of evaluating $\int \sin x \cos x \mathrm{d}x$, the integral that appeared on the right hand side of the differential equation. There are many valid approaches, either by direct integration or by using a double angle formula or using a substitution and it was surprisingly that none of these came to mind for many candidates.

(b) Only a handful of candidates made any meaningful progress with this request and the majority offered very little as an attempt at a solution. A common error was to substitute $y = \mathrm{p}(x)$ into equation (1) only partially, with $\frac{\mathrm{d}y}{\mathrm{d}x}$ retained. A similar procedure for equation (2) led to inevitable confusion between y, p and c.

(c)(i) Many candidates missed the obvious approach here, expressed clearly in the instruction to *verify*. All that was necessary was to differentiate the given function and substitute into equation (3). A surprising number of candidates attempted to solve equation (3), not realising that this was not within their scope, and that the structure of the question was to lead them through a method of solution.

(ii) Many candidates had abandoned their attempts at this question by this stage, but to those who proceeded, this was familiar territory.

(iii) This part brought together parts (b) and (c) but only a few candidates grasped the structure of the question and pursued it to the end.

4 Simultaneous linear differential equations

(i) Almost all candidates gained the majority of the marks in this part. The method of approach was understood and any marks lost were due to arithmetic slips in manipulating the functions or in solving the auxiliary equation.

(ii) Again, candidates knew what they had to do. Answers to part (i) were followed through for the majority of the marks.

(iii) The method was applied successfully, although by this stage most candidates had made at least one arithmetical slip. Fully correct expressions for x and y were rarely seen.

(iv) Candidates who had made errors in the previous parts of this question were at a disadvantage because their expression for $x + y$ did not usually tend to a finite limit.

Credit was given for the correct interpretation of the behaviour of the candidate's incorrect expression.